Model Questions for Differential Calculus.

MCQ Questions

1) The n^{th} differentiation for 'cot x' is :-

a) $\text{cosec}^2 x$ b) $-\text{cosec}^2 x$

c) $\text{cosec}\, x \cot x$ d) $-\text{cosec}\, x \cot x$

2) The n^{th} differentiation for $\sec^{-1} x$ is :-

a) $\frac{-1}{\sqrt{x^2-1}}$ b) $\frac{1}{\sqrt{x^2-1}}$

c) $\frac{-1}{x\sqrt{x^2-1}}$ d) $\frac{1}{x\sqrt{x^2-1}}$

3) The n^{th} derivative for x^m is (for $m>n$) :-

a) $n!$ b) $m!$

c) $\frac{m!}{(m-n)!}x^{m-n}$ d) $\frac{n!}{(m-n)!}x^{m-n}$

4) The n^{th} derivative for $\frac{1}{x+a}$ is :-

a) $\frac{(-1)^n n!}{(x+a)^{n+1}}$ b) $\frac{(-1)^n n!}{(x+a)^n}$

c) $\frac{(-1)^n (n+1)!}{(x+a)^{n+1}}$ d) $\frac{(-1)^{n+1} n!}{(x+a)^n}$

5) The n^{th} derivative for $\cos(ax+b)$ is :-

a) $a^n \sin(n\pi/2 + ax+b)$ b) $a^n \cos(n\pi/2 + ax+b)$

c) $b^n \sin(n\pi/2 + ax+b)$ d) $b^n \cos(n\pi/2 + ax+b)$

6) The n^{th} derivative for $e^{ax}\sin bx$ is :-

a) $(a^2+b^2)^{n/2} e^{ax} \sin(bx + n\tan^{-1} b/a)$

b) $(a^2+b^2)^n e^{ax} \cos(bx + n\tan^{-1} b/a)$

c) $(a^2+b^2)^n e^{ax} \sin(bx + n\tan^{-1} b/a)$

d) $(a^2+b^2)^{n/2} e^{ax} \cos(bx + n\tan^{-1} b/a)$

7) The n^{th} derivative for $\tan^{-1} x$ is

a) $(-1)^{n-1} (n-1)! \sin^n\theta \sin n\theta$, $\theta = \tan^{-1} a/x$

b) $(-1)^{n-1} (n-1)! \sin^n\theta \sin n\theta$, $\theta = \cot^{-1} a/x$

c) $(-1)^{n-1} (n-1)! \sin^n\theta \sin n\theta$, $\theta = \tan^{-1} x$

d) $(-1)^{n-1} (n-1)! \sin^n\theta \sin n\theta$, $\theta = \cot^{-1} x$.

8) Expansion of e^x is :-

a) $1 + x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$

b) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$

c) $1 - x + \frac{x^3}{3!} - \frac{x^5}{5!} + \dots$

d) $1 + x - \frac{x^2}{2!} + \frac{x^3}{3!} - \frac{x^4}{4!} + \dots$

9) Expansion of $\sin x$ is :-

a) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$

b) $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$

c) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$

d) $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$

10) Expansion of $\tan^{-1} x$ is :-

a) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$

b) $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$

c) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$

d) $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$

Short Answer type Questions

1) If $y = \sin(m\sin^{-1}x)$. Prove that
$$(1-x^2)y_2 - xy_1 + m^2y = 0$$

2) If $y = (\sin^{-1}x)^2$. Prove that
$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$$

3) Expand $e^x \cos x$ as far as terms containing x^4

4) Expand $\frac{e^x}{1+e^x}$ by Maclaurin's theorem as far as x^4.

Long Answer type Questions

1) Prove that $\frac{d^3x}{dy^3} = -\frac{\left(\frac{d^3y}{dx^3}\cdot\frac{dy}{dx} - 3\left(\frac{d^2y}{dx^2}\right)^2\right)}{\left(\frac{dy}{dx}\right)^5}$

2) & find y_n if $y = \frac{1}{a^2+x^2}$

3) State and prove Leibnitz theorem.

4) State and prove Maclaurins theorem and write the conditions of failure for Taylor's Series.